Total Printed Pages : **16**

Booklet No. 11272

M.Phil./Ph.D. ADMISSION TEST, 2018

Paper II

Subject : 131 - MATHEMATICS

Roll No. (In figures) ....(In words) ....

OMR Sheet Sr. No. ....

Signatures of Invigilators 1. .... 2. ....

Names of Invigilators 1. .... 2. ....

Time : 2 Hours Max. Marks : 200

GENERAL INSTRUCTIONS

1. Read the instructions given on the Question Booklet and OMR Sheet before starting the answers. All the entries should be filled by **blue or black ball point pen**.

2. The Question Booklet contains **100** questions and all questions are compulsory.

3. Each question is of **2** marks. There is **no negative marking**.

4. Candidates must ensure that the Question Booklet issued to them has all the questions. Defective Question Booklet can be got changed within **10** minutes.

1. प्रश्नों के उत्तर लिखने से पूर्व प्रश्न-पुस्तिका और ओ.एम.आर. शीट पर दिये हुए निर्देश पढ़ें। सभी प्रविष्टियाँ **नीले अथवा काले बॉल पॉइन्ट पेन** से भरें।

2. प्रश्न-पुस्तिका में **100** प्रश्न हैं और सभी प्रश्न अनिवार्य हैं।

3. प्रत्येक प्रश्न **2** अंक का है। कोई **नकारात्मक अंकन (negative marking) नहीं होगा।**

4. परीक्षार्थी सुनिश्चित कर लें कि उन्हें जो प्रश्न-पुस्तिका दी गई है उसमें सभी प्रश्न अंकित हैं। त्रुटिपूर्ण प्रश्न-पुस्तिका **10** मिनट की अवधि में बदलवाई जा सकती है।

5. In case of any discrepancy between English and Hindi versions of a question, English version will be taken as correct, wherever there are both versions.

6. Select and darken the circle corresponding to the answer [(A) or (B) or (C) or (D)] in OMR sheet.

7. In case more than one circles are darkened in a question, it will not be evaluated.

8. Do not make any stray marks on OMR sheet and do not fold it.

9. Any candidate found removing pages from the Question Booklet may be disqualified and prosecuted.

10. Use of unfair means will disqualify the candidate from the examination.

11. Cell phone, calculator or any such devices are not allowed in the Examination Hall.

12. No candidate is allowed to leave the seat before handing over the original OMR sheet to the invigilator. Candidate can take Question Booklet and Carbon copy of OMR sheet.

5. किसी प्रश्न के अंग्रेजी और हिन्दी रूपान्तरणों में भिन्नता होने की स्थिति में अंग्रेजी रूपान्तरण सही माना जायेगा जहाँ प्रश्न-पत्र दोनों भाषाओं में है।

6. सही उत्तर का चयन करें तथा सम्बन्धित [(A) अथवा (B) अथवा (C) अथवा (D)] गोले को ओ.एम.आर. शीट में काला करें।

7. किसी प्रश्न में एक से अधिक गोले को काला करने पर उसे जाँचा नहीं जायेगा।

8. ओ.एम.आर. शीट पर किसी तरह का चिह्न न बनायें और न ही उसे मोड़ें।

9. प्रश्न-पुस्तिका से पृष्ठ निकालते हुए पाये जाने पर परीक्षार्थी को अयोग्य घोषित किया जा सकता है और उसके विरुद्ध विधिक कार्यवाही भी की जा सकती है।

10. अनुचित साधनों का उपयोग करने पर परीक्षार्थी को परीक्षा के लिए अयोग्य घोषित कर दिया जायेगा।

11. सेलफोन, संगणक और ऐसी किसी भी अन्य प्रविधियों को परीक्षा भवन में लाने की अनुमति नहीं हैं।

12. ओ.एम.आर. शीट की मूल प्रति वीक्षक को सुपुर्द किये बिना किसी भी परीक्षार्थी को अपना स्थान छोड़ने की अनुमति नहीं है। परीक्षार्थी प्रश्न-पुस्तिका एवं ओ.एम.आर. शीट की कार्बन प्रति को अपने साथ ले जा सकेगा।

1. The number of non-empty subsets of a set consisting of 8 elements is :

(A) 256

(B) 255

(C) 128

(D) 127

2. If A and B are subsets of a set X, then $[A \cap (X-B)] \cup B$ is equal to :

(A) $A \cup B$

(B) $A \cap B$

(C) A

(D) B

3. The sequence $\langle S_n \rangle$ where $S_n = \left(1 + \frac{2}{n}\right)^{n+3}$ converges to :

(A) e

(B) e^2

(C) $e+3$

(D) e^2+3

4. Set of rational numbers :

(A) a finite set

(B) has two limit points only

(C) is not a field

(D) is an integral domain

5. For the given sequence $\langle (-1)^n \rangle$ $\forall\, n \in N$, which one of the following statements is **correct** ?

(A) Limit superior = Limit inferior

(B) Neither limit superior nor limit inferior exists

(C) Limit superior = 1 and Limit inferior = − 1

(D) Limit superior = 1 and Limit inferior = 0

6. Which of the following statements is **true** ?

(A) For any positive number E, there is a natural number n such that $\frac{1}{n} < E$.

(B) Between any two real numbers there is no irrational number.

(C) Convergent sequence is not bounded.

(D) None of the above is true.

7. Which of the following set is compact ?

(A) N, set of natural numbers

(B) Q, set of rational numbers

(C) $[a, \infty[$

(D) $[a, b]$

8. Which of the following set is uncountable ?

(A) {1, 4, 9, 16, 25,..........}

(B) $\{2n : n \in N\}$

(C) all rational numbers

(D) all irrational numbers

9. Σu_n is the series of positive terms and $\lim_{n\to\infty} (u_n)^{1/n} > 1$ then the series is :

(A) Divergent

(B) Convergent

(C) Oscillatory

(D) None of these

10. If $f(x) = x^2$ $\forall\, x \in R$, f is :

(A) not continuous on R

(B) uniformly continuous on R

(C) not uniformly continuous on R

(D) none of these

29. Let V be a vector space over a field F. Then which one of the following mapping is called a linear functional ?

(A) $f: V \to V$

(B) $f: F \to F$

(C) $f: V \to F$

(D) $f: F \to V$

30. Let $T: U \to V$ be a surjective linear mapping and $\dim U = 6$, $\dim V = 3$ then :

(A) $\dim \text{Ker } T > 4$

(B) $\dim \text{Ker } T = 4$

(C) $\dim \text{Ker } T = 3$

(D) $\dim \text{Ker } T > 3$

31. Let $V = R^2$ with standard inner product. Then which of the following tuples of elements of V forms an orthonormal basis ?

(A) $\{(1, -1), (-1, -1)\}$

(B) $\{(-1, 0), (0, -1)\}$

(C) $\{(1, 0), (0, 1), (1, 1)\}$

(D) $\{(-1, 1), (1, 1)\}$

32. Every finite dimensional inner product space has always :

(A) an orthogonal basis but not orthonormal basis

(B) an orthonormal basis

(C) an orthonormal basis but not orthogonal basis

(D) neither orthogonal nor orthonormal basis

33. An orthonormal set is always :

(A) linearly independent

(B) linearly dependent

(C) both linearly dependent and independent

(D) none of these

34. In how many ways can a mixed double Tennis game be arranged from 9 married couples, if no husband and wife play in the same game ?

(A) ${}^9P_2 \times {}^7P_2$

(B) $\frac{1}{2}\left({}^9P_2 \times {}^7P_2\right)$

(C) 1512

(D) 756

35. How many triangles can be formed by joining 15 points when 7 of them are in the same straight line ?

(A) ${}^{22}C_3 - {}^7C_3$

(B) ${}^{15}C_3 + {}^7C_3$

(C) ${}^{22}P_3 - {}^7P_3$

(D) 420

36. 45 candidates appear in a competitive examination; then there will be at least n candidates whose roll numbers differ by a multiple of 44, find n.

(A) 1

(B) 2

(C) 3

(D) 4

37. Find the number of Mathematics students at a college taking at least one of the languages French, German and Russian, given that : 42 study Russian, 45 study German, 65 study French, 20 study French and German, 25 study French and Russian, 15 study German and Russian, and 8 study all the three languages.

(A) 124

(B) 162

(C) 92

(D) 100

38. n books are distributed among n students, and after the return of books, these are redistributed to same n students. In how many ways can the books be redistributed so that no student gets the same book again ?

(A) $(n!)^2\left[1 - \frac{1}{1!} + \frac{1}{2!} - \ldots + (-1)^n \frac{1}{n!}\right]$

(B) $(n!)\left[1 - \frac{1}{1!} + \frac{1}{2!} - \ldots + (-1)^n \frac{1}{n!}\right]$

(C) $(n!)^2\left[1 + \frac{1}{1!} + \frac{1}{2!} + \ldots + (-1)^n \frac{1}{n!}\right]$

(D) $(n!)^2\left[1 + \frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^n \frac{1}{n!}\right]$

39. The number of divisors of 6480 are :

(A) 40

(B) 50

(C) 60

(D) 30

40. Fill the blank by **correct** choice.

There are ________ primes of the form $(4n+3)$, where n is a natural number.

(A) exactly n^2

(B) eleven

(C) finite number of

(D) infinitely many

41. Every number and its cube when divided by m leaves the same remainder, then m is :

(A) 6

(B) 7

(C) 11

(D) 13

42. If p is a prime number, then

$(a+b)^p = a^p + b^p + \lambda$,

where λ is a multiple of :

(A) $(p+1)$

(B) $(p-1)$

(C) p

(D) p!

43. Let $\phi(n)$ is to denote Euler's function, then $\phi(18)$ equals :

(A) 6!

(B) 6

(C) 18!

(D) $\phi(6) \cdot \phi(3)$

44. For a positive integer m, a primitive root of m is a positive integer b such that g.c.d. $(b, m) = \lambda$, and the order b modulo m is $\phi(m)$, then λ is :

(A) 1

(B) b

(C) bm

(D) m

45. Let the set Z of all integers under the binary operation $*$ be defined by $\alpha * \beta = \alpha + \beta + 1$, for all $\alpha, \beta \in Z$, then the **true** statement is :

(A) $(\alpha * \beta) * \gamma \neq \alpha * (\beta * \gamma)$

(B) 0 is the identity element

(C) inverse of α is $(-2-\alpha)$

(D) $(Z, *)$ forms a non-abelian group

46. Let there be a set of non-negative integers modulo 5 under the operation of addition modulo 5. Then which of the following is **not** true ?

(A) 0 is the identity

(B) inverse element exists

(C) commutative property is not satisfied

(D) the addition is associative

47. The statement which is **not** true is :

(A) Every finite cyclic group of order n is isomorphic to $(Z_n, +_n)$.

(B) Every infinite cyclic group is isomorphic to $(Z, +)$.

(C) Any two cyclic groups of the same order are isomorphic.

(D) The relation of isomorphism in the set of groups is not an equivalence relation.

48. Let $H_1 = \{0, \pm 2, \pm 4, \pm 6, ...\}$,

$H_2 = \{0, \pm 3, \pm 6, \pm 9, ...\}$,

and $Z = \{0, \pm 1, \pm 2, \pm 3, ...\}$,

then ________ is not a subgroup of $(Z, +)$.

Fill the blank space **correctly** by :

(A) $(H_1 \cup H_2, +)$

(B) $(H_1, +)$

(C) $(H_2, +)$

(D) $(H_1 \cap H_2, +)$

49. The statement which is **not** true is :

(A) The system $(Z, +, \cdot)$ is an integral domain.

(B) $(2Z, +, \cdot)$ is a commutative ring.

(C) The system $(Z_6, +_6, \odot_6)$ is not a ring.

(D) The system $(Z_6, +_6, \odot_6)$ is a commutative ring with identity element 1.

50. The statement which is **false**, is :

(A) The system (C, +, •) is a field.

(B) Every field is an integral domain.

(C) Every finite integral domain is a field.

(D) The ring $(Z_n, +_n, \odot_n)$ is a field if and only if n is not a prime number.

51. Let ϕ be a homomorphism from the ring (R, +, •) into the ring (R_1, +, •), then which of the following statement is **not** true ?

(A) $\phi(0) = 0'$, where $0'$ is additive identity in R_1

(B) ϕ is one-to-one if and only if $\ker(\phi) \neq \{0\}$

(C) $(\ker(\phi), +, \cdot)$ is an ideal of (R, +, •)

(D) $(\phi(R), +, \cdot)$ is a subring of $(R_1, +, \cdot)$

52. In which quadrant $\left(\frac{1+2i}{1-i}\right)$ is situated ?

(A) First

(B) Second

(C) Third

(D) Fourth

53. The complex number z having least magnitude and satisfying $|z-5i| \leq 3$ will be :

(A) $\frac{12}{5} + \frac{16i}{5}$

(B) $-12+16i$

(C) $\frac{12}{25} + \frac{16i}{25}$

(D) $\frac{16}{5} - \frac{12i}{5}$

54. Let $\left(0 < \theta < \frac{\pi}{2}\right)$, then the nature of roots of equation $x^2 - 2x\sin\theta + 1 = 0$, is :

(A) pure imaginary

(B) real

(C) equal

(D) complex

55. The range of $y = \cosh x$ is :

(A) $(1, \infty)$

(B) $[1, \infty]$

(C) $[1, \infty)$

(D) $[1, 0]$

56. $\text{cosech}\left(\frac{i\pi}{2}\right) - \cosh(i\pi)$ equals :

(A) $1-i$

(B) $1+i$

(C) $-1-i$

(D) $-1+i$

57. If $\tan(\alpha + i\beta) = ik$, $|k| \neq 1$, then α will be :

(A) k

(B) 0

(C) $-k$

(D) $1+k^2$

58. The **true** statement for the function $[f(z) = xy + iy]$ is :

(A) Analytic everywhere.

(B) Continuous in first quadrant only.

(C) Everywhere continuous but not analytic.

(D) No where continuous but Cauchy-Riemann equations are satisfied.

59. Let C be a rectifiable arc joining the points $z = a$ and $z = b$; then the **true** statement is :

(A) $\int_C dz$ = Arc length of C

(B) $\int_C |dz| = (b - a)$

(C) $\int_C z dz = \frac{1}{3}(b^3 - a^3)$

(D) $\int_C |dz|$ = Arc length of C

60. From $z=0$ to $z=4+2i$ along the curve C given by $z=t^2+it$, the value of $\int_C \bar{z}\, dz$ is :

(A) 10

(B) $-\frac{8i}{3}$

(C) $10+\frac{8}{3}i$

(D) $10-\frac{8i}{3}$

61. The curve C being $|z|=1$, the value of $\int_C \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} dz$ is $\frac{21\pi i}{\lambda}$, then λ is :

(A) 32

(B) 16

(C) 8

(D) 4

62. Expansion of $\log(1+z)$ about $z=0$ is $z-\frac{z^2}{2}+\frac{z^3}{3}-...+(-1)^m\cdot\frac{z^n}{n}+...$, then m is :

(A) n−1

(B) n+1

(C) n

(D) 2n−1

63. The Laurent's series expansion of $f(z)=\frac{z^2-4}{(z+1)(z+4)}$ valid for $|z|<1$ is $f(z)=-1+\sum_{n=1}^{\infty}(-1)^m(1+4^{-n})\cdot z^n$, then m is :

(A) n

(B) n+1

(C) n−1

(D) −n

64. The residue of $\frac{z^2-2z}{(z+1)^2(z^2+4)}$ at $z=-1$ is :

(A) 1

(B) −1

(C) $\frac{14}{25}$

(D) $\frac{-14}{25}$

65. The transformation $\omega=\frac{1+z}{1-z}$ transforms $|\omega|\leq 1$ into :

(A) half plane $Re(z)\geqslant 0$

(B) half plane $Re(z)\leq 0$

(C) half plane $Im(z)\leq 0$

(D) half plane $Im(z)\geqslant 0$

66. The transformation which maps the points $z=2, 1, 0$ into the points $\omega=1, 0, i$ respectively is :

(A) $\omega=\frac{1}{z-i}$

(B) $\omega=\frac{2(z-1)}{z+i(2-z)}$

(C) $\omega=\frac{4(z-1)}{z+i(2-z)}$

(D) $\omega=\frac{4(z-1)}{z-i(2+z)}$

67. The solution of the initial value problem $\frac{dy}{dx}=\frac{3}{2}y^{1/3}, y(0)=0$ is :

(A) Solution exists and is unique

(B) Solution exists but is not unique

(C) Solution does not exists

(D) None of these

68. The singular solution of the following differential equation $(px-y)^2 = p^2 - 1$ is :

(A) $x^2 - y^2 = 1$

(B) $x^2 + y^2 = 1$

(C) $y^2 - x^2 = 1$

(D) None of these

69. The complementary function of the following differential equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$ is :

(A) $(C_1 + C_2 x)\log x$

(B) $(C_1 + C_2 x^{-1})\log x$

(C) $(C_1 + C_2 \log x)x^{-1}$

(D) $(C_1 + C_2 \log x)x$

70. The solution of the boundary value problem $\frac{d^2y}{dx^2} + y = 0$ with $y(0) = 0$, $y(1) = 0$ is :

(A) $y(x) = 0$

(B) $y(x) = 1$

(C) $y(x) = -1$

(D) None of these

71. The general solution of the equation $(y+z)p + (z+x)q = (x+y)$ is :

(A) $\phi\left[\frac{y-x}{y-z}, (x+y)^2 (x+y+z)^2\right] = 0$

(B) $\phi\left[\frac{x-y}{y-z}, (x-y)^2 (x+y+z)\right] = 0$

(C) $\phi\left[\frac{x-y}{z-y}, (x-y)(x+y+z)\right] = 0$

(D) None of these

72. The complete solution of $z = pq$ is :

(A) $2\sqrt{(az)} = ax + by + c$

(B) $az\sqrt{2} = ax - by + c$

(C) $\sqrt{2} = ax + by + cz$

(D) None of these

73. The solution of $r = 6x$ is :

(A) $z = y^3 + x^2 f(x) + \phi(x)$

(B) $z = x^3 + x f(y) + \phi(y)$

(C) $z = x^2 - y^2 f(x) + \phi(y)$

(D) None of these

74. The Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ in cylindrical polar co-ordinates (r, θ, z) is :

(A) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$

(B) $\frac{1}{r^2}\frac{\partial^2 u}{\partial r^2} - \frac{1}{r}\frac{\partial u}{\partial r} - \frac{1}{r}\frac{\partial^2 u}{\partial \theta^2} - \frac{\partial^2 u}{\partial z^2} = 0$

(C) $r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} - \frac{\partial^2 u}{\partial \theta^2} - \frac{\partial^2 u}{\partial z^2} = 0$

(D) None of these

75. The solution of the following partial differential equation $\frac{\partial^2 z}{\partial x \partial y} = 2(x+y)$ is :

(A) $z = xy + x^2y^2 + \phi_1(x) + \phi_2(y)$

(B) $z = x^2y + xy^2 + \phi_1(y) + \phi_2(x)$

(C) $z = xy + x^2y + \phi_1(y) + \phi_2(x)$

(D) None of these

76. The general solution of the following partial differential equation $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2\,\partial y} + \frac{2\partial^3 z}{\partial x\,\partial y^2} = 0$ is :

(A) $z = \phi_1(y+x) + \phi_2(y-x) + \phi_3(y+3x)$

(B) $z = \phi_1(2y+x) + \phi_2(y-2x) + \phi_3(y+x)$

(C) $z = \phi_1(y+x) + \phi_2(y+2x) + \phi_3(y-2x)$

(D) $z = \phi_1(y) + \phi_2(y+x) + \phi_3(y+2x)$

77. The subsidiary equation for the following Linear equation

$Pp + Qq = R$ [Where P, Q, R are functions of x, y, z] is :

(A) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

(B) $\frac{dx}{Q} = \frac{dy}{R} = \frac{dz}{P}$

(C) $\frac{dx}{R} = \frac{dy}{P} = \frac{dz}{Q}$

(D) None of these

78. For the polynomial $P(x) = 2x^3 - 6x + 3$, P''' is :

(A) 9

(B) 10

(C) 11

(D) 12

79. Evaluate $\Delta^n[ax^n + bx^{n-1}]$:

(A) $a\underline{|n}$

(B) $b\underline{|n^2}$

(C) an

(D) bn^2

80. Simpson's Three-Eight rule is :

(A) $\int_{x_0}^{x_0+nh} y\,dx \simeq \frac{3h}{8}\,[(y_0+y_1) + 3(y_2+y_3+\ldots + y_{n-1}) + 2(y_4+y_5+\ldots+y_{n-3})]$

(B) $\int_{x_0}^{x_0+nh} y\,dx \simeq \frac{3h}{8}\,[(y_0+y_2) + 2(y_3+y_4+\ldots + y_{n-1}) + 3(y_4+y_5+\ldots+y_{n-3})]$

(C) $\int_{x_0}^{x_0+nh} y\,dx \simeq \frac{3h}{8}\,[(y_0+y_3) + 3(y_4+5+\ldots + y_{n-1}) + 2(y_5+y_6+\ldots+y_{n-3})]$

(D) $\int_{x_0}^{x_0+nh} y\,dx \simeq \frac{3h}{8}\,[(y_0+y_n) + 3(y_1+y_2+y_4 + y_5+\ldots+y_{n-1}) + 2(y_3+y_6+\ldots+y_{n-3})]$

81. Evaluate $\sum_{m=0}^{\infty} \frac{1}{(10+m)^2}$:

(A) $\frac{1}{10} + \frac{1}{2}\cdot\frac{1}{10^2} + \frac{1}{12}\,\frac{2}{10^3} - \frac{1}{720}\,\frac{24}{105}$

(B) $\frac{1}{10^2} + \frac{1}{3}\,\frac{1}{10^3} + \frac{1}{13}\,\frac{2}{10^4} - \frac{1}{721}\,\frac{25}{106}$

(C) $\frac{1}{10^3} + \frac{1}{4}\,\frac{1}{10^4} + \frac{1}{14}\,\frac{2}{10^5} - \frac{1}{722}\,\frac{26}{107}$

(D) $\frac{1}{10^4} + \frac{1}{5}\,\frac{1}{10^5} + \frac{1}{15}\,\frac{2}{10^6} - \frac{1}{723}\,\frac{27}{108}$

82. If a function $\phi[f(x)]$ having a variation achieves a maximum or minimum at $y = y_0(x)$, where $y(x)$ is an interior point of the domain of definition of the functional then at $y = y_0(x)$:

(A) $\delta\phi = 0$

(B) $\delta\phi = 1$

(C) $\delta\phi = 2$

(D) $\delta\phi = 3$

83. The extremal of the functional $\phi[y(x)] = \int_0^1 (1+y''^2)\,dx$; $y(0)=0, y'(0)=1, y(1)=1, y'(1)=1$ lies on :

(A) $y = mx + \frac{a}{m}$

(B) $y = mx + c$

(C) $y = x$

(D) $y = -x^2$

84. A curve of minimum length connecting two points on a surface $F(x, y, z) = 0$, the minimum of the functional $l = \int_{x=x_0}^{x_1} \sqrt{1 + y'^2 + z'^2}\,dx$, the function $y(x)$ and $z(x)$ are subject to the condition :

(A) $F(x, y, z) = 0$

(B) $F(x, y) = 0$

(C) $F(y, z) = 0$

(D) $F(z, x) = 0$

85. If for every continuous function $\eta(x)$, $\int_a^b \xi(x)\,\eta(x)\,dx = 0$, where $\xi(x)$ is continuous in [a, b], then :

(A) $\xi(x) = 1$ on [a, b]

(B) $\xi(x) = -1$ on [a, b]

(C) $\xi(x) = 0$ on [a, b]

(D) None of these

86. The necessary condition for $\int_{x_1}^{x_2} f(x, y, y')dx$ sto be an extremum is :

(A) $\delta \int_{x_1}^{x_2} f(x, y, y')dx = 0$

(B) $\delta \int_{x_1}^{x_2} f(x, y, y')dx \neq 0$

(C) $\delta \int_{x_1}^{x_2} f(x, y, y')dx = 1$

(D) $\delta \int_{x_1}^{x_2} f(x, y, y')dx = -1$

87. An integral equation

$$\alpha(x)\,g(x) = f(x) + \lambda \int_a^x k(x, t)\,g(t)dt,$$

where a is constant, upper limit of integration is variable, $f(x)$, $\alpha(x)$ and $k(x, t)$ are known function, λ is a non-zero real or complex parameter. The integral equation is Volterra's integral equation of first kind when :

(A) $\alpha \equiv 1$

(B) $\alpha \equiv -1$

(C) $\alpha \equiv 0$

(D) None of these

88. The following integral equation

$$\alpha(x)\,g(x) = f(x) + \lambda \int_a^x k(x, t)\,g(t)dt,$$

where a is constant, $f(x)$, $\alpha(x)$, $k(x, t)$ are known function while $g(x)$ is unknown function, λ is a non-zero real or complex parameter and upper limit of integration is variable is homogeneous Volterra's integral equation of second kind when :

(A) $\alpha \equiv 1, f(x) \equiv 0$

(B) $\alpha \not\equiv 1, f(x) \not\equiv 0$

(C) $\alpha \equiv 2, f(x) \not\equiv 0$

(D) None of these

89. The following integral equation

$$\alpha(x)\, g(x) = f(x) + \lambda \int_a^b k(x, t)\, g(t) dt,$$

where a and b are both constants, $f(x)$, $\alpha(x)$, $k(x, t)$ are known functions while $g(x)$ is unknown function with λ is a non-zero real or complex parameter is Fredholm integral equation of first kind when :

(A) $\alpha \equiv 1$

(B) $\alpha \equiv -1$

(C) $\alpha \equiv 0$

(D) None of these

90. The following integral equation

$$\alpha(x)\, g(x) = f(x) + \lambda \int_a^b k(x, t)\, g(t) dt,$$

where a and b are both constants, $f(x)$, $\alpha(x)$, $k(x, t)$ are known functions while $g(x)$ is unknown function with λ a non-zero real or complex is homogeneous Fredholm integral equation of second kind when :

(A) $\alpha \not\equiv 1, f(x) \not\equiv 0$

(B) $\alpha \equiv 1, f(x) \equiv 0$

(C) $\alpha \not\equiv 2, f(x) \not\equiv 2$

(D) $\alpha \equiv -1, f(x) \not\equiv 0$

91. Let θ, ϕ, Ψ be the generalized co-ordinates of a system, then the Cartesian coordinates (x, y, z) of any point of it at anytime t can be expressed as :

(A) $x = x(t, \theta, \phi, \Psi)$
$y = y(t, \theta, \phi, \Psi)$
$z = z(t, \theta, \phi, \Psi)$

(B) $x = x(t, \dot{\theta}, \phi, \Psi)$
$y = y(t, \theta, \dot{\phi}, \Psi)$
$z = z(t, \theta, \phi, \dot{\Psi})$

(C) $x = x(t, \theta, \dot{\phi}, \Psi)$
$y = y(t, \theta, \phi, \dot{\Psi})$
$z = z(t, \dot{\theta}, \phi, \Psi)$

(D) $x = x(\dot{t}, \theta, \phi, \Psi)$
$y = y(\dot{t}, \dot{\theta}, \phi, \Psi)$
$z = z(\dot{t}, \theta, \dot{\phi}, \Psi)$

92. The intersection of two convex set is :

(A) A convex set

(B) Not a convex set

(C) Not always convex set

(D) None of these

93. If at any iteration of the simplex algorithm we find $z_j - c_j < 0$, for at least one j and for this j, $y_{ij} \leq 0\ \forall i = 1, \ldots, m$ then if the objective function is to be maximized, $y_{ij} \leq 0\ \forall i = 1, 2, \ldots, m$ admits :

(A) an optimal solution

(B) a bounded solution

(C) an unbounded solution

(D) none of these

94. If X be any feasible solution to the primal viz. Min. $Zp = Cx$ s.t. $Ax \geqslant b$ and $x \geqslant 0$ and ω is any feasible solution to the dual then :

(A) $Zp \geqslant ZD$

(B) $Zp \leq ZD$

(C) $ZD \geqslant Zp$

(D) None of these

95. The dual of the dual a given primal is :

(A) Dual

(B) Primal

(C) Optimal solution

(D) None of these

96. The Laplace transform of a function F(t), for $t \geq 0$ is a function of a new variable p given by the relation $L\{F(t)\} = \int_0^\infty e^{-pt} F(t)\, dt$ exists when :

(A) converges for some value of p

(B) diverges for some value of p

(C) oscillatory for some value of p

(D) none of these

97. If $\bar{f}(p)$ is the Fourier transform of $f(x)$ then the Fourier transform of $f(ax)$ is :

(A) $\bar{f}\left(\frac{p}{a}\right)$

(B) $\frac{1}{a}\bar{f}(p)$

(C) $\frac{1}{p}\bar{f}\left(\frac{a}{p}\right)$

(D) $\frac{1}{a}\bar{f}\left(\frac{p}{a}\right)$

98. The length of the circular helix $\bar{r}(u) = a \cos ut + a \sin uj + cuk$ from (a, 0, 0) to (a, 0, 2πc) is :

(A) $2\pi\sqrt{a^2 + c^2}$

(B) $3\pi\sqrt{a^2 - c^2}$

(C) $4\pi\sqrt{c^2 - a^2}$

(D) $5\pi\sqrt{a + c}$

99. The necessary and sufficient condition for a curve to be a straight line is [at all points of the curve].

(A) Curvature $x = 1$

(B) Curvature $x = -1$

(C) Curvature $x \neq 1$

(D) Curvature $x = 0$

100. The covariant differentiation of the sum (or difference) of two tensors is the sum (or difference) of their :

(A) Covariant derivatives

(B) Invariant derivatives

(C) Partial derivatives

(D) None of these

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